Ch 15 Modern Phys 3300 HW

Note Title

15.5 $E = 15.0\ eV$ $V = 5.0\ eV$

What L gives no reflection $\longrightarrow T = 1$

$T = 1$ when $K_{II} L = n\pi$ (see expressions 15.1)

$$L = \frac{n\pi}{K_{II}}$$

$$= n\pi \frac{\hbar}{\sqrt{2m(E-V)}}$$

$$= n * 1.94 * 10^{-10} m$$

6) max reflection for T min

maximize sin term

$\rightarrow$ when $K_{II} L = \frac{m\pi}{2}$ $m = 1, 3, \ldots$

or

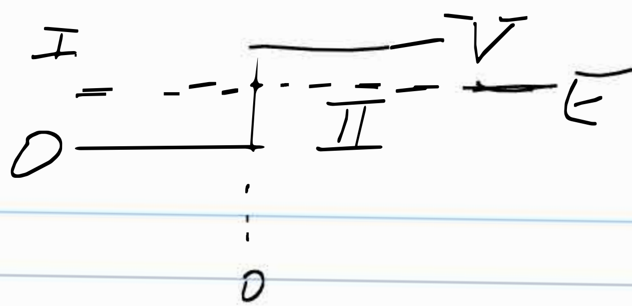
$\frac{m}{2} = n - \frac{1}{2}$

for $n = 1, 2, 3 \ldots$

$$L_{mins} = \left(n - \frac{1}{2}\right) \frac{\pi}{2} \frac{\hbar}{\sqrt{2m(E-V)}}$$

$$= \left(n - \frac{1}{2}\right) 1.94 * 10^{-10} m$$

15.9



$$\psi_{II} = Ce^{\gamma x} + De^{-\gamma x}$$

regardless of ψ_{I} , ψ_{II} must be finite in entire region II

$\rightarrow$ $C = 0$

15.18

$$\psi_1 = \left(\frac{\alpha}{4\pi}\right)^{1/4} (2\sqrt{\alpha})(x)\, e^{-\alpha x^2/2} \qquad \alpha = \frac{\omega_c m}{\hbar}$$

a) $$\int_{-\infty}^{\infty} \psi_1^* \psi_1 \, dx = \left(\frac{\alpha}{4\pi}\right)^{1/2} (4)\,\alpha \left(\int_0^{\infty} x^2 e^{-\alpha x^2} dx\right) * 2$$

LOOK UP !

$$= \left(\frac{\alpha}{4\pi}\right)^{1/2} 8\alpha \left(\frac{1}{2^2 \alpha}\right) \sqrt{\frac{\pi}{\alpha}} = 1$$

b) sketch $\psi^* \psi$ prob density vs ... x

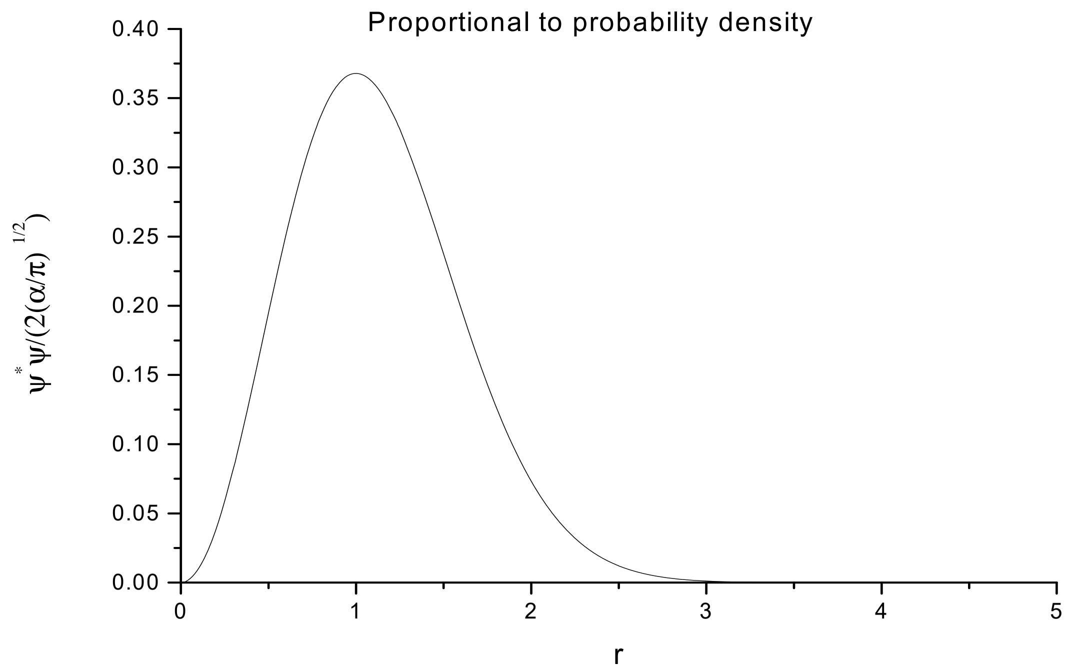
Make dimensionless — note α has dim of $\frac{1}{(\text{length})^2}$

so

$$r = \sqrt{\alpha}\, x$$

Plot

$$\left(\frac{\psi^* \psi}{4 \left(\frac{\alpha}{4\pi} \right)^{1/2}} \right) = r^2 e^{-r^2}$$



c) Prob of $x \to +$ (or $-$) is $1/2$ $\longrightarrow$ symmetry

15.19 $\psi_0 = Ae^{-bx^2}$ find b, plug into S.E.

$$\frac{d\psi_0}{dx} = -2bx\,\underbrace{Ae^{-bx^2}}_{\psi_0}$$

$$\frac{d^2\psi}{dx^2} = -2bxA\left(e^{-bx^2}(-2bx)\right) + \left(-2bAe^{-bx^2}\right)(1)$$

note ψ_0 in here

so

$$-\frac{\hbar^2}{2m}\left(-2bA\psi_0\right)\left(1-2bx^2\right) + \frac{1}{2}m\omega_c^2x^2\psi_0 = E\psi_0$$

gather terms, linearly indep for x^n terms for $n=0$, $n=2$

$$\frac{\hbar^2 b}{m} = E_0$$

and $\frac{\hbar^2 b}{m}(-2b) + \frac{1}{2} m\omega_c^2 = 0$

$$b = \frac{1}{2} m\omega_c / \hbar$$

b) Normalize

$$2\int_0^\infty A^2 e^{-2bx^2} dx = 1 \qquad (\text{CRC \# } 663)$$

$$2 \frac{A^2\sqrt{\pi}}{2\sqrt{2b}} = 1 \qquad A = \left(\frac{2m\omega_c}{h}\right)^{\frac{1}{4}}$$

c) $E_0 = \frac{\hbar^2 b}{m} = \frac{\hbar\omega_c}{2}$ $\quad n = 0$ ... as expected